

<b>Vertical shift up <math>k</math> units:</b>	$F(x)=f(x)+k$
<b>Vertical shift down <math>k</math> units:</b>	$F(x)=f(x)-k$

## Example 1

Sketch the graph of  $g(x)=\sqrt{x}+4$

Solution:

$y=\sqrt{x}$       Basic function

$y=\sqrt{x}+4$       Vertical shift up 4 units

Begin with the basic function defined by  $f(x)=\sqrt{x}$  and shift the graph up 4 units.

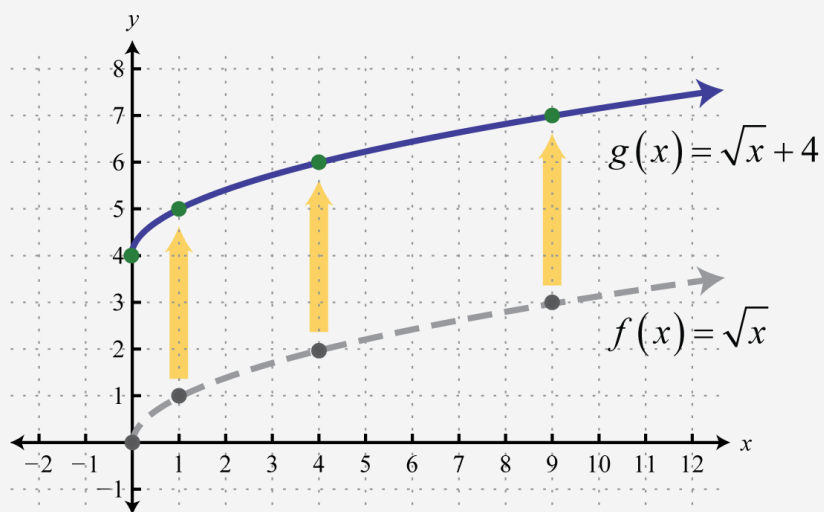
x   y   y+4

0   0   4

1   1   5

4   2   6

Answer:



<b>Horizontal shift left <math>h</math> units:</b>	$F(x)=f(x+h)$
<b>Horizontal shift right <math>h</math> units:</b>	$F(x)=f(x-h)$

## Example 2

Sketch the graph of  $g(x)=(x-4)^3$ .

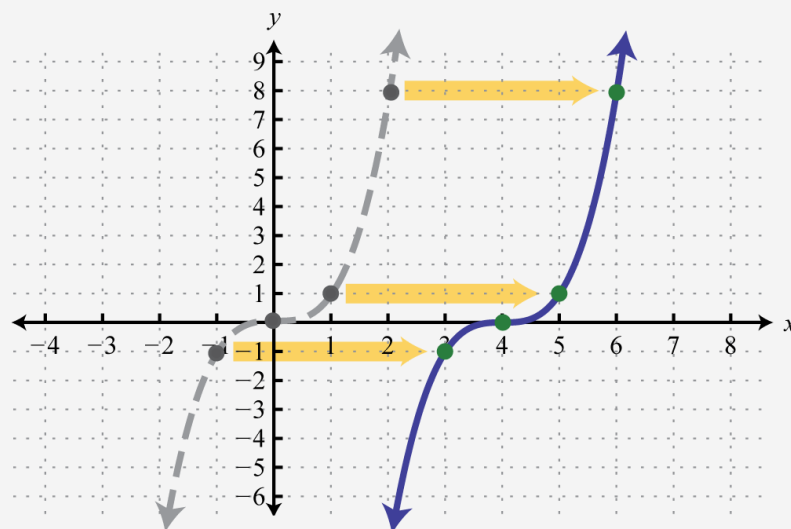
Solution:

$y=x^3$       Basic function

$y=(x-4)^3$       Horizontal shift right 4 units

Begin with a basic cubing function defined by  $f(x)=x^3$  and shift the graph 4 units to the right.

Answer:



## Example 3

Sketch the graph of  $g(x)=|x+3|-5$ .

Solution:

Start with the absolute value function and apply the following transformations.

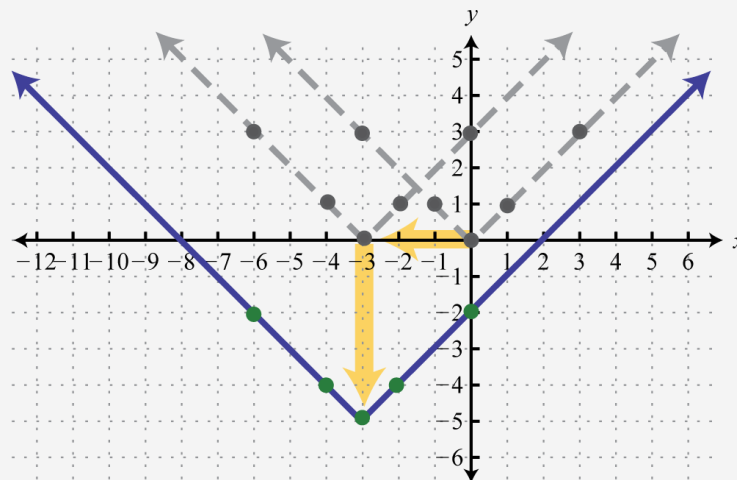
$y=|x|$       **Basic function**

$y=|x+3|$       **Horizontal shift left 3 units**

$y=|x+3|-5$       **Vertical shift down 5 units**

$x-3$	$x$	$y$	$y-5$
-4	-1	1	-4
-3	0	0	-5
-2	1	1	-4

Answer:



### Example 4

Sketch the graph of  $g(x)=\frac{1}{x-5}+3$

Solution:

Begin with the reciprocal function and identify the translations.

$$y = \frac{1}{x}$$

Basic function

$$y = \frac{1}{x-5}$$

Horizontal shift right 5 units

$$y = \frac{1}{x-5} + 3$$

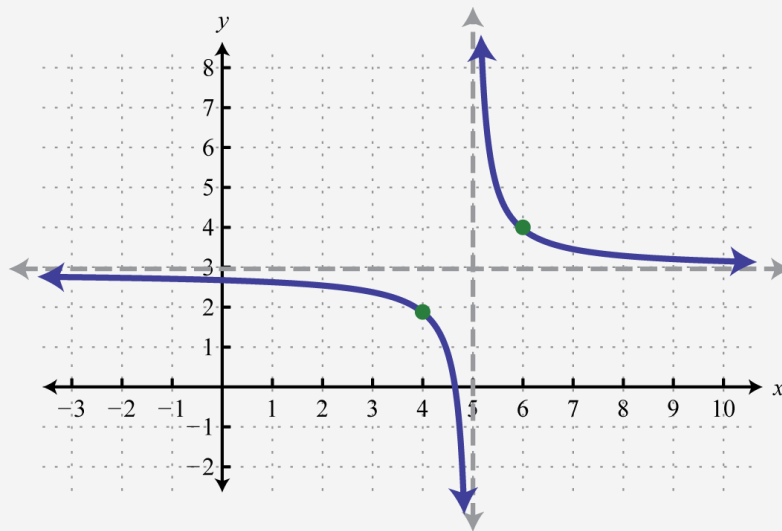
Vertical shift up 3 units

$x+5$	$x$	$y$	$y+3$
4	-1	-1	2
5	0	0	3
6	1	1	4

5      0      0      3 ----→ new asymptotes  $x=5$ ,  $y=3$

Take care to shift the vertical asymptote from the  $y$ -axis 5 units to the right and shift the horizontal asymptote from the  $x$ -axis up 3 units.

Answer:



<b>Reflection about the <math>y</math>-axis:</b>	$F(x)=f(-x)$
<b>Reflection about the <math>x</math>-axis:</b>	$F(x)=-f(x)$

When sketching graphs that involve a reflection, consider the reflection first and then apply the vertical and/or horizontal translations.

## Example 5

Sketch the graph of  $g(x) = -(x + 5)^2 + 3$ .

Solution:

Begin with the squaring function and then identify the transformations starting with any reflections.

$y = x^2$	Basic function
$y = -x^2$	Reflection about the x-axis
$y = -(x + 5)^2$	Horizontal shift left 5 units
$y = -(x + 5)^2 + 3$	Vertical shift up 3 units

$x-5$	$x$	$y$	$-y+3$
-6	-1	1	2
-5	0	0	3
-4	1	1	2

Use these translations to sketch the graph.

Answer:

