Inverses

* they are when x & y are switched

$$(x, y) \hookrightarrow (y, x)$$

* reflected over $y = x$ line

* domain of $f(x)$ = range $f^{-1}(x)$
  range of $f(x)$ = domain $f^{-1}(x)$

algebrically find $f^{-1}(x)$

* switch x & y the solve for y.

ex 1 $f(x) = 3x - 4$

$y = 3x - 4$
$x = 3y - 4$
$x + 4 = 3y$
$\frac{x+4}{3} = y$

$f^{-1}(x) = \frac{x+4}{3}$

ex 2 $f(x) = \frac{x+5}{2}$

$y = \frac{x+5}{2}$
$x = \frac{y+5}{2}$
$2x - 5 = y$

$f^{-1}(x) = 2x - 5$

ex 3 $f(x) = \frac{5}{x+4}$

$x = \frac{5}{y+4}$
$x(y+4) = 5$
$y + 4 = \frac{5}{x}$
$y = \frac{5}{x} - 4$

$f^{-1}(x) = \frac{5}{x} - 4$

prove inverses

$f(g(x)) = x$ & $g(f(x)) = x$

ex 4 prove $f(x) = 2x - 1$ & $f^{-1}(x) = \frac{x+1}{2}$

$f(g(x)) = 2\left(\frac{x+1}{2}\right) - 1 = x + 1 - 1 = x$

$g(f(x)) = \frac{2x - 1 + 1}{2} = \frac{2x}{2} = x$

✓ yes, inverses