Complex Numbers Notes

What is i and how does it work?

$i = \sqrt{-1}$ imaginary #s

$\sqrt{-36} = \underbrace{\sqrt{36}}_{6}\underbrace{\sqrt{-1}}_{i} = 6i$

$\sqrt{-40} = \underbrace{\sqrt{4}}_{2}\sqrt{10}\underbrace{\sqrt{-1}}_{i} = 2i\sqrt{10}$

The rotation:

$\frac{exp}{4} \rightarrow$ Remain

R1 $i = \sqrt{-1} = i$

R2 $i^2 = (\sqrt{-1})^2 = -1$

R3 $i^3 = (\sqrt{-1})^3 = \underbrace{\sqrt{-1}\sqrt{-1}}_{-1}\underbrace{\sqrt{-1}}_{i} = -i$

noR $i^4 = \underbrace{\sqrt{-1}\sqrt{-1}}_{-1}\underbrace{\sqrt{-1}\sqrt{-1}}_{-1} = 1$

1. $i^9 = i^{-1}$

$\frac{9}{4} \rightarrow$ R1

2. $i^{35} = -i$

$\frac{35}{4} \rightarrow$ R3

Solving for x

* square root to solve $\rightarrow \pm$
* always reduce!

3. $\sqrt{x^2} = \sqrt{36}$

$\boxed{x = \pm 6}$

4. $x^2 - 40 = 0$

$\sqrt{x^2} = \sqrt{40}$

$x = \pm\sqrt{40} \rightarrow \underbrace{\sqrt{4}}_{2}\sqrt{10}$

$\boxed{x = \pm 2\sqrt{10}}$

5. $x^2 + 25 = 0$

$\sqrt{x^2} = \sqrt{-25} \rightarrow \underbrace{\sqrt{25}}_{5}\underbrace{\sqrt{-1}}_{i}$

$\boxed{x = \pm 5i}$

6. $(x-5)^2 + 10 = -26$

$\sqrt{(x-5)^2} = \sqrt{-36}$

$x - 5 = \pm 6i$

$\boxed{x = 5 \pm 6i}$

OR $x = \pm 6i + 5$

<u>Complex Number</u>: $a + bi$

real # imaginary #

Adding/Subtracting

* combo like terms
* $x \sim i$

7. $(3+7i)+(-5-6i)$

$-2 + i$

8. $2i - 5 - 6i - 9$

$-4i - 14$

9. $2(5-4i)+(7-3i)$

$= 10 - 8i + 7 - 3i$

$= \boxed{17 - 11i}$

10. $(3-11i)-(7-8i)$

$= 3 - 11i - 7 + 8i$

$= \boxed{-4 - 3i}$

11. $4(12+4i)-6(-1-i)$

$= 48 + 16i + 6 + 6i$

$= \boxed{54 + 22i}$

Multiply

* ( )( ) → FOIL
* $i^2 = -1$

12. $2i(4i-3)$

$= 8i^2 - 6i$

replace $i^2=-1$ $= 8(-1) - 6i$

$= \boxed{-8-6i}$

13. $(6-5i)(4+7i)$

$= 24 + 42i - 20i - 35i^2$

$= 24 + 22i \underbrace{- 35(-1)}_{+35}$

$= \boxed{59 + 22i}$

14. $(4+9i)(4-9i)$

$= 16 \cancel{-36i + 36i} - 81i^2$

$= 16 - 81(-1)$

$= \boxed{97}$

Divide (Rationalize)

goal: no i's in denominator

complex conjugate: switch sign of i

$(3-2i)(3+2i)$ CC

15. $\frac{5}{3i}\left(\frac{-3i}{-3i}\right)$

$= \frac{-15i}{-9i^2} = \frac{-15i}{-9(-1)} = \frac{-15i}{9}$

$= \boxed{\frac{-5i}{3}}$

16. $\frac{-4}{7i}\left(\frac{-7i}{-7i}\right) = \frac{28i}{-49i^2} = \frac{28i}{-49(-1)}$

$= \frac{28i}{49} = \boxed{\frac{4i}{7}}$

17. $\frac{4}{3+i}\left(\frac{3-i}{3-i}\right)$

$\frac{12-4i}{9 \cancel{-3i+3i} - i^2} = \frac{12-4i}{9+1} = \frac{12-4i}{10} = \boxed{\frac{6-2i}{5}}$

FOIL (-1) reduce combo divide all by 2

18. $\frac{2-3i}{6+i}\left(\frac{6-i}{6-i}\right)$

always denom.

$\frac{12-2i-18i+3i^2}{36 \cancel{-6i+6i} - i^2} = \frac{12-20i+3(-1)}{36+1} = \boxed{\frac{9-20i}{37}}$