

CONCEPT SUMMARY Transformations of Functions

For a function $f(x)$, the graph of $f(x) = a \cdot f[b(x - h)] + k$ represents a transformation of the graph of that function by translation, reflection, or stretching.

WORDS	EQUATIONS	GRAPHS
Horizontal translation of f right 2 units (altering h)	$f(x)$ becomes $g(x) = f(x - 2)$ $f(x) = x^2 + x$ $g(x) = (x - 2)^2 + (x - 2) = x^2 - 3x + 2$	
Vertical translation of f up 3 units (altering k)	$f(x)$ becomes $h(x) = f(x) + 3$ $f(x) = x^2 + x$ $h(x) = x^2 + x + 3$	
Reflection of f across the x-axis (altering a)	$f(x)$ becomes $-f(x)$ $f(x) = x^2 + x$ $-f(x) = -(x^2 + x) = -x^2 - x$	
Reflection of f across the y-axis (altering b)	$f(x)$ becomes $f(-x)$ $f(x) = x^2 + x$ $f(-x) = (-x)^2 + (-x) = x^2 - x$	
Horizontal stretch of f by a factor of 2 (altering b)	$f(x)$ becomes $f(\frac{1}{2}x)$ $f(x) = x^2 + x$ $f(\frac{1}{2}x) = (\frac{1}{2}x)^2 + \frac{1}{2}x = \frac{1}{4}x^2 + \frac{1}{2}x$	
Vertical stretch of f by a factor of 2 (altering a)	$f(x)$ becomes $2f(x)$ $f(x) = x^2 + x$ $2f(x) = 2(x^2 + x) = 2x^2 + 2x$	

Do You UNDERSTAND?

- ESSENTIAL QUESTION** What do the differences between the equation of a function and the equation of its parent function tell you about the differences in the graphs of the two functions?
- Reason** Do k and h affect the input or output for $g(x) = f(x) + k$ and $g(x) = f(x - h)$? Explain. © MP.2
- Error Analysis** Margo is comparing the functions $f(x) = |x|$ and $g(x) = |x + 1| - 5$. She said the graph of g is a vertical translation of the graph of f 5 units down and a horizontal translation of the graph of f 1 unit right. What is Margo's error? © MP.3

Do You KNOW HOW?

- Graph each function and its parent function.
- $g(x) = |x| - 1$
 - $g(x) = -|x|$
 - $g(x) = x^2 - 2$
 - $g(x) = 4x$
 - $g(x) = (x - 3)^2$
 - $g(x) = -x$
 - $g(x) = \frac{1}{2}|x|$
 - $g(x) = |5x|$
 - Which types of transformations in Exercises 4–11 do not change the shape of a graph? Which types of transformations change the shape of a graph? Explain.

PRACTICE & PROBLEM SOLVING

Scan for
Multimedia



Practice

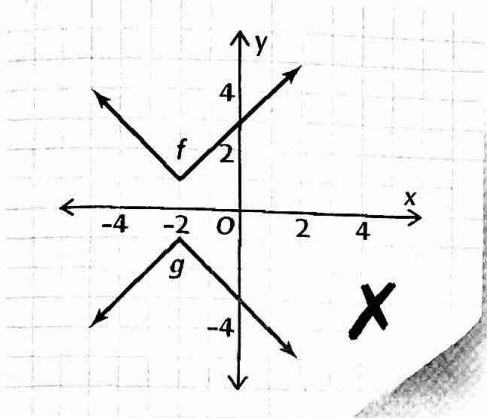


Tutorial

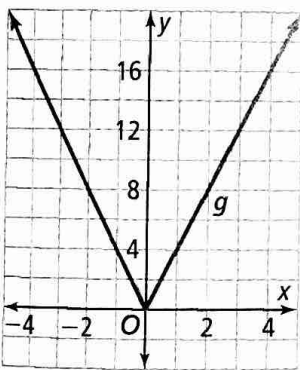
Additional Exercises Available Online

UNDERSTAND

13. Use Structure Write a function g with the parent function $f(x) = x^2$ that has a vertex at $(3, -6)$. © MP.7
14. Error Analysis Describe and correct the error a student made in graphing $g(x) = f(-x)$ as a reflection across the y -axis of the graph of $f(x) = |x + 2| + 1$. © MP.3



15. Higher Order Thinking Describe the transformation g of $f(x) = |x|$ as a stretch and as a compression. Then write two equations to represent the function. What can you conclude? Explain.



16. Use Structure The graph of the parent function $f(x) = x^2$ is reflected across the y -axis. Write an equation for the function g after the reflection. Show your work. Based on your equation, what happens to the graph? Explain. © MP.7
17. Error Analysis Monisha is comparing $f(x) = |x|$ and $g(x) = |2x - 4|$. She said the graph of g is a horizontal translation of the graph of f 4 units to the right and a horizontal compression of the graph of f by a factor of 2. What is Monisha's error? © MP.3

PRACTICE

Graph each function as a translation of its parent function, f . How did the transformation affect the domain and range? SEE EXAMPLE 1

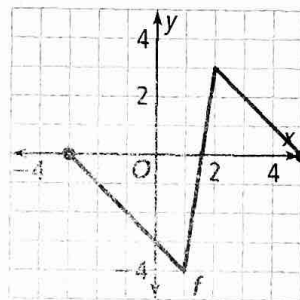
18. $g(x) = |x| - 5$ 19. $g(x) = (x + 1)^2$
20. $g(x) = |x - 3|$ 21. $g(x) = x^2 + 2$

What is the equation for the image graph? Check by graphing. SEE EXAMPLE 2

22. Reflect $f(x) = x^2 + 1$ across the x -axis.
23. Reflect $f(x) = x^2 + 1$ across the y -axis.

Graph each function as a vertical stretch or compression of the parent function $f(x) = |x|$. SEE EXAMPLE 3

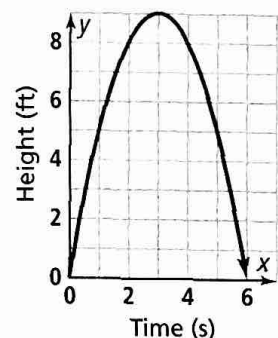
24. $g(x) = 0.25|x|$ 25. $g(x) = 3x^2$
26. $g(x) = 1.5|x|$ 27. $g(x) = 0.75x^2$
28. Use the graph of $f(x)$ to graph $y = f(x + 1) + 2$. SEE EXAMPLE 4



What transformations of $f(x) = x^2$ are applied to the function g ? SEE EXAMPLE 5

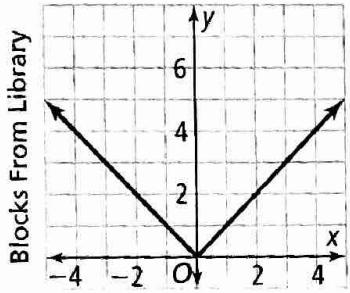
29. $g(x) = 2(x + 1)^2$ 30. $g(x) = (x - 3)^2 + 5$
31. $g(x) = -x^2 - 6$ 32. $g(x) = 4(x - 7)^2 - 9$

33. The graph shows the height y in feet of a flying insect x seconds after taking off from the ground. Write an equation that represents the height of the insect as a function of time. SEE EXAMPLE 6



APPLY

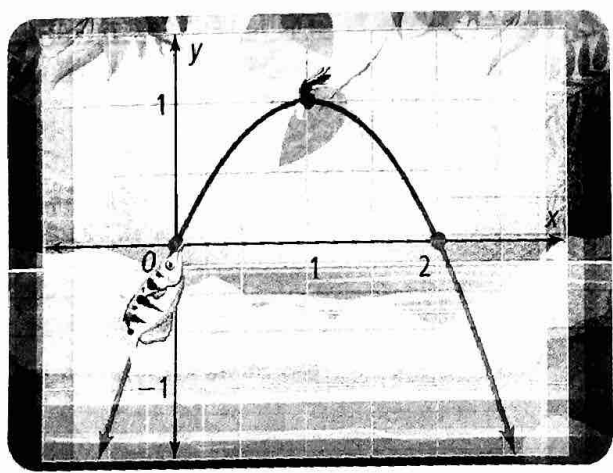
34. Model With Mathematics Chiang walks to school each day. She passes the library halfway on her walk to school. She walks at a rate of 1 block per minute. The graph shows the distance Chiang is from the library as she walks to school. © MP.4



Minutes Before Passing Library Minutes After Passing Library

- Write a function, f , to model the distance Chiang is from the library when she walks to school.
- If Chiang jogs to school, she travels at a rate of 2.5 blocks per minute. Write a function, g , to model the distance Chiang is from the library when she jogs to school.
- Graph the function, g , that models the distance Chiang is from the library when she jogs to school.

35. Model With Mathematics The archer fish spits water at flying insects to knock them into the water. The path of the water is shown with x and y distances in feet. Write an equation to represent the path of the water in relation to the coordinate grid. Then determine the coordinates of the point of maximum height of the water. © MP.4



ASSESSMENT PRACTICE

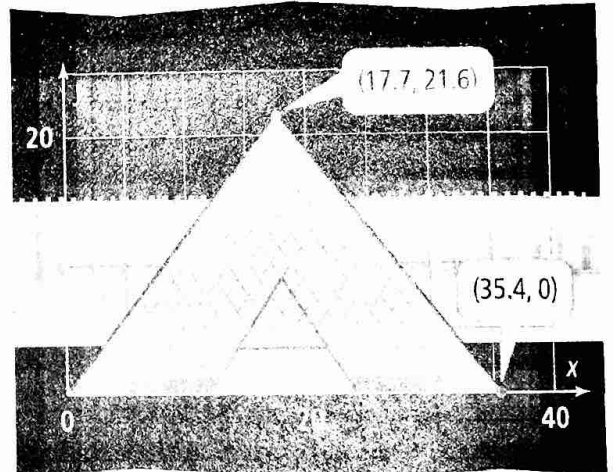
36. Match each equation on the left to an equation on the right that has the same translation of $y = |x|$.

- | | |
|-----------------------|----------------------|
| A. $y = x - 1$ | D. $y = x + 1 - 1$ |
| B. $y = - x + 1 $ | E. $y = 2 x - 1$ |
| C. $y = - x + 1 - 1$ | F. $y = x + 1 $ |

37. SAT/ACT Which translation is part of transforming $f(x) = x^2$ into $h(x) = (x + 4)^2 - 2$?

- Ⓐ left 4 units Ⓒ right 2 units
Ⓑ left 2 units Ⓓ right 4 units

38. Performance Task The Louvre Pyramid in Paris is shown on the coordinate grid, where x and y are measured in meters and the ground is represented by the x -axis.



Part A The outline of the Pyramid is a transformation of the function $f(x) = |x|$. Write a function g to model the outline of the Pyramid.

Part B What is the domain and range of the function that models the outline of the Pyramid? What do the domain and range represent?