10/27 Notes

properties of exponents (see p266)

① $x^n \cdot x^m = x^{n+m}$

ex $x^3 \cdot x^5 = x^8$

② $\frac{x^n}{x^m} = x^{n-m}$

ex $\frac{x^5}{x^7} = x^{5-7} = x^{-2}$

③ $(x^n)^m = x^{n \cdot m}$

ex $(x^3)^4 = x^{3 \cdot 4} = x^{12}$

④ $x^{-n} = \frac{1}{x^n}$ & $\frac{1}{x^{-n}} = x^n$

ex $2^{-4} = \frac{1}{2^4}$ & $\frac{3}{2^{-3}} = 3(2)^3$

⑤ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

ex $\left(\frac{x}{y}\right)^4 = \frac{x^4}{y^4}$

class examples

write, simplify & make all exponents positive.

1. $x^{-6} = \frac{1}{x^6}$

2. $(2x^2)^3 = 2^3 x^{2 \cdot 3} = 8x^6$

* do not forget the #s too! *

3. $\frac{x^5 y}{x^4 y^3} = \boxed{\frac{x}{y^2}}$

* I like to split with a dashed line, the variables *

4. $\left(\frac{2x^2y^4}{6x^5y}\right)^3 = \left(\frac{y^3}{3x^3}\right)^3 = \boxed{\frac{y^9}{27x^9}}$

$\frac{1}{3} x^{-3} y^3$

* first reduce the inside, then deal with the exponent *

5. $\left(\frac{7x^5y^2z^8}{10x^2y^6z^2}\right)^{-3} = \left(\frac{7x^3z^6}{10y^4}\right)^{-3}$

$\frac{7}{10} x^3 y^{-4} z^6$

* similar to #4 at first but the negative flips the fraction, seen below *

$\left(\frac{10y^4}{7x^3z^6}\right)^3 = \boxed{\frac{1000y^{12}}{343x^9z^{18}}}$

Exponent Skills Practice

Definitions and Notation

- **Positive exponent:** represents repeated multiplication $x^n = x \cdot x \cdot \cdots \cdot x$ (n x's in all)
- **Zero exponent:** always equal to 1 for all $x \neq 0$. $x^0 = 1$
- **Negative exponent:** represents a fraction $x^{-n} = \frac{1}{x^n}$ or $\frac{1}{x^{-n}} = x^n$
- **Fractional exponent:** power over root $x^{a/b} = (\sqrt[b]{x})^a = \sqrt[b]{x^a}$

Rules for Calculating and Simplifying

- **Multiplying powers with the same base:** add the exponents $a^m \cdot a^n = a^{m+n}$

  Real number example: $2^3 \cdot 2^5 = 2^8$.

- **Dividing powers with the same base:** subtract the exponents $\frac{a^m}{a^n} = a^{m-n}$

  Real number example: $\frac{6^{10}}{6^2} = 6^8$

- **Power of a power:** multiply the exponents: $(a^m)^n = a^{mn}$

  Real number example: $(3^2)^4 = 3^8$.

- **Multiplying powers with different bases, same exponent:** combine and keep the same exponent $a^n \cdot b^n = (ab)^n$

  Real number example: $2^4 \cdot 5^4 = (2 \cdot 5)^4 = 10^4$.

  Another real number example: $\left(\frac{3}{4}\right)^2 = \frac{3^2}{4^2} = \frac{9}{16}$.

It is important to be able to use these rules in either direction.

Investigation • Properties of Exponents

Name________________________________ **Period**__________ **Date**____________

Use expanded form to review and generalize the properties of exponents.

Step 1 Write each product in expanded form, and then rewrite it in exponential form.

a. $2^3 \cdot 2^4$ **b.** $x^5 \cdot x^{12}$ **c.** $10^2 \cdot 10^5$

Step 2 Generalize your results from Step 1. $a^m \cdot a^n =$ __________

Step 3 Write the numerator and denominator of each quotient in expanded form. Reduce by eliminating common factors, and then rewrite the factors that remain in exponential form.

a. $\frac{4^5}{4^2}$ **b.** $\frac{x^8}{x^6}$ **c.** $\frac{(0.94)^{15}}{(0.94)^5}$

Step 4 Generalize your results from Step 3. $\frac{a^m}{a^n} =$ __________

Step 5 Write each quotient in expanded form, reduce, and rewrite in exponential form.

a. $\frac{2^3}{2^4}$ **b.** $\frac{4^5}{4^7}$ **c.** $\frac{x^3}{x^8}$

Step 6 Rewrite each quotient in Step 5 using the property you discovered in Step 4.

Step 7 Generalize your results from Steps 5 and 6. $\frac{1}{a^n} =$ __________

Investigation • Properties of Exponents (continued)

Step 8 Write several expressions in the form $(a^n)^m$. Expand each expression, and then rewrite it in exponential form. Generalize your results.

Step 9 Write several expressions in the form $(a \cdot b)^n$. Don't multiply a times b. Expand each expression, and then rewrite it in exponential form. Generalize your results.

Step 10 Show that $a^0 = 1$, using the properties you have discovered. Write at least two exponential expressions to support your explanation.